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## New Progressing Cavity Pump (NPCP) for Multiphase and Viscous Liquid Production

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### Abstract

This paper presents a new pump concept, called the *New Progressing Cavity Pump* (NPCP). The NPCP is composed of a Progressing Cavity Pump (PCP) and a system of Hydraulic Regulators (HR) installed inside the pump in between the cavities. The HR are self-regulated devices that recirculate the fluid between the cavities in order to control the pump thermo-hydraulic response and to avoid excessive built up of heat, which might result in premature failure of the pump's stator.

It is common knowledge that the traditional Progressing Cavity Pumps (PCP) have shown significant problems when dealing with multiphase mixtures, with high gas void fraction (GVF). We propose a new concept, NPCP, which is able to handle much better such conditions, as its internal hydraulic regulators (HR) recirculate fluid inside the pump. The benefits are multiple. First, it uniformizes the pressure across the pump length, which stabilizes the temperature. Second, it compensates the compressed gas volume of progressing cavities. Furthermore, it protects the stator and therefore, improves the pump's performance.

The relatively small dimensions of hydraulic regulators HR and their distribution along the pump result in an efficient multiphase design. Compared to traditional PCP, the new NPCP system better controls the reliability parameters, such as temperature and friction torque, reduces energy consumption and enhances hydraulic performance (flow rate, delivered pressure).

Several industrial NPCP have been manufactured and tested in liquid, both water and oil, as well in multiphase flow (air and liquid). This paper presents and describes the bench test results and shows that NPCP improves both reliability and hydraulic performance over existing pumps.

### Introduction

The recent developments of new production systems clearly indicates that there are considerable industrial interests in pumping multiphase fluids, in particular the combination of high gas volume and oil or water, or small amounts of wax and sand. Traditional Progressing Cavity Pump (PCP) often faces up with reliability problems, mostly because of gas phase compression causing heat to build up, resulting in premature failure of elastomeric stator and pump disfunctioning.

We have recently published a study that analyzes the behavior of traditional PCP in multiphase flow (see [1]) and the reasons for traditional PCP poor performance are presented. The *thermo-hydraulic* process is the result of fundamental gas laws, which explains that as the pressure increases inside a cavity with constant volume, the temperature does increase too. Typical pressure distribution along the PCP is the result of gas volume compression and slippage flow between the rotor and the stator. Close to the end of the discharge, the gas compresses and the volume is compensated by the slippage flow. Multiple tests have shown that a disproportionate amount of pressure is developed by the discharge stages, which causes excessive heat build-up. Another *thermo-mechanical process* occurs as a result of disproportionate pressure gradient between two contiguous cavities. Because of differential pressure, the stator material is strained inside the low pressure cavity. Thus, the compression stress of the rotor increases the friction torque (viscous) and the increasing temperature becomes a measure of rotor-stator viscous-friction torque. In multiphase flow conditions, the excessive pressure gradient in the discharge stages causes both *thermo-hydraulic* and *thermo-mechanical* physical processes which reduce the pump reliability and performance. Furthermore, the traditional PCP reliability is related to pressure distribution which also depends upon the pump design and production conditions, i.e. gas flow rate, delivered pressure and rotational velocity. Among the multiphase pump designs, there are two major classes:

- Class 1: Pumps that keep the actual PCP design and adapt the compression fit between the rotor and stator, which is gradually reduced with the distance from suction end, as developed by Mirza K and Wild A [2]. Slipping flows introduce a trade-off between delivering pressure and flow rate requirement, and compensating

the compressed volume of gas cavities. This design trade-off is difficult to control.

- Class 2: The New PCP (NPCP), which includes the *hydraulic regulators* (HR) installed inside the rotor between the cavities, the function of which is to control the reliability parameters, i.e. pressure distribution and the developed temperature as well as the viscous torque.

The NPCP have two distinct types of flows:

- The HR recirculation flow, which is internal to the cavities. The hydraulic regulators re-circulate fluid between cavities, with no flow rate loss, which results in a well balanced thermo-hydraulic and mechanical processes.
- The external slippage of the original PCP.

This paper describes the NPCP system: (1) the HR fluid recirculation principle; (2) the design method that complies with standard reliability specifications (temperature and frictional torque); and (3) the pump performance in a production environment (that is: gas void fraction GVF, flow rate, delivered pressure, power). We present experimental results, which show NPCP's ability to handle two-phase flow with high void fraction (GVF between 0 and 0.9), and a real level of production: rotation speed,  $N$ , between 200 and 400 rpm and pressure,  $P$ , between 0 and 40 bars. The experimental program, presented in this paper, was performed at PCM's facility test rig and CREMHyG research laboratory in Grenoble, France.

## NPCP Design Configuration

The NPCP is the result of *hydraulic regulators* HR installed inside the rotor of actual PCP, between the cavities. The design of HR (distribution and dimensions) allows for pump control of multiphase flow (i.e., pressure distribution, temperature build-up and viscous torque) and therefore improves reliability parameters and pump lift performance. The operating mode of NPCP is "new" in the sense that the cavities are inter-connected through self-regulated HR, instead of individual sealed cavities of traditional PCP. Therefore, the thermo-hydraulic-mechanical behavior of NPCP is completely different and due to HR the pump performance and reliability are under control.

A typical example of HR setting up on the rotor is shown in Figure 1A and the cavities internal recirculation operation through HR is represented in Figure 1B. In this application the HR is a circular channel, carefully shaped to transfer designed fluid volume and pressure. In order to control HR regulation process, valves or nozzles can be added.

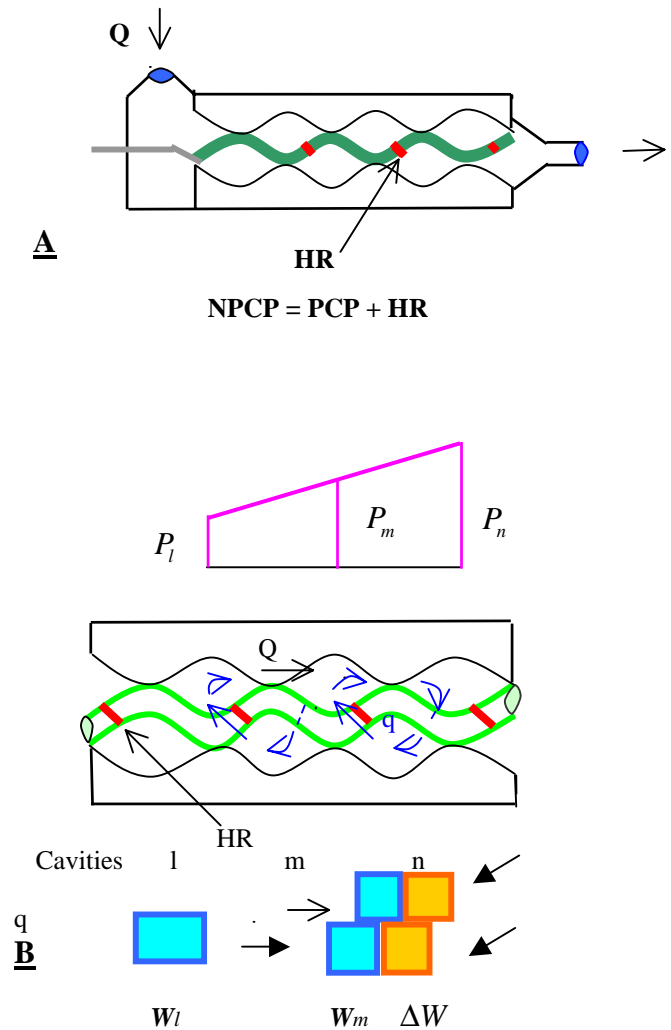
Consider three cavities  $l$ ,  $m$  and  $n$  (Figure 1B), the pressure distribution  $P_l < P_m < P_n$  and the pumped flow rate  $Q$ .

The flow rate  $Q$  moves from  $l$  to  $m$ , then gas volume is compressed ( $P_m > P_l$ ) inside the cavity  $m$ . Due to the pressure difference between the cavities  $n$  and  $m$ , the HR will transfer a fluid flow inside the cavity  $m$  (flowing from  $n$  to  $m$ ) which compensates the compressed gas volume and balance the pressure difference. At the same time, as the cavities' volume

is constant, this transfer will balance the pressure  $P_n$  and  $P_m$ . Therefore, this is an iterative process: pressure difference involves fluid transfer, which balance the pressure distribution and compensate the gas compressed volume. The HR recirculation flow is self-regulated, i.e. while the pressure is balanced the flow rate  $q$  is reduced.

The total flow rate ( $Q+q$ ) moves forward to the cavity  $n$ , the recirculated  $q$  flows back to  $m$  while the pumped  $Q$  will continue to the next stage.

During the moving, the gas is compressed and the rate of compression is a function of differential pressure (polytropic compression [1]); so the transferred  $q$  flow rate will compensate the compressed gas volume.



**Figure 1. NPCP design layout (A) and HR recirculation between the cavities ( $l, m, n$ ) (B).**

The system efficiency is depending on the HR design: dimensions and distribution along the pump. This can be expressed in terms of thermodynamic gas law and hydraulic compensation.

Suppose that the gas of a cavity volume  $W_G$  with a mass  $M_G = \rho_G \cdot W_G$ . The gas is compressed at the pressure  $P$  and

develops the temperature  $T$ . We can write the gas law as follows:

$$\frac{P}{M_G \times T} \approx C \quad (\text{Eq. 1})$$

As the cavity moves toward the pump discharge, the pressure  $P$  increases. For a constant gas mass  $M_G$ , the temperature  $T$  builds-up. If the pressure gradient  $P$  is balanced by an added mass (i.e. the ratio pressure / mass becomes roughly constant), then the temperature can be maintained near the initial level. As we outlined in a previous paper [1], based on the Eq.1 , the polytropic gas compression becomes:

$$\frac{P^{\frac{2k-1}{k}}}{\rho_G^k \times T} = c \quad (\text{Eq. 2})$$

where  $k$  is defined as to be the polytropic coefficient .

Assuming the volume of cavity  $l$  moves to cavity  $m$ , without any recirculation  $q$ , the gas conservation masses (Eq. 3) and polytropic compression (Eq. 4) become:

$$M_l = \rho_l \times W_l \quad M_m = \rho_m \times W_m \quad M_l = M_m \quad (\text{Eq. 3})$$

$$\frac{P_l}{\rho_l^k} = \frac{P_m}{\rho_m^k} \quad (\text{Eq. 4})$$

We then find the compressed gas volume ( Figure 1 B ) as:

$$\Delta W = W_l - W_m = \alpha \times W \times \left[ 1 - \left( \frac{P_l}{P_m} \right)^{\frac{1}{k}} \right] \quad (\text{Eq. 5})$$

$$W_l = \alpha \times W$$

defining  $\alpha$  =GVF to be the gas void fraction . Therefore, the recirculation flow rate  $q$  will compensate the compressed volume  $\Delta W$  , as defined in Eq.5 and balance the pressure gradient:

$$q = \omega \times \frac{1}{\sqrt{\zeta}} \left( 2 \cdot \frac{\Delta P}{\rho} \right)^{1/2} \quad (\text{Eq.6})$$

$$\Delta P = P_n - P_m$$

$$\Delta W = n \times \int q \cdot dt \approx n \times t \times q$$

where  $\omega$  is the HR flow section ,  $\zeta$  is the pressure drop coefficient through HR ,  $\Delta P$  is the differential pressure of HR ,  $t$  is the hydraulic regulator HR working time and  $n$  is the number of hydraulic regulators ( the HR distribution ). Indeed, as the cavities are moving  $t$  is the time during which the cavity  $n$  will transfer  $q$  to the cavity  $m$  (i.e. the cavities are connected by hydraulic regulators HR ).

In practice, optimum design of hydraulic regulators HR system (dimension, distribution) is the result of an iterative process, as shown in Figure 2 :

1. Assume a given pressure distribution ( $P_l, P_m, P_n$ )
2. Get the compressed gas volume  $\Delta W$  , as a function of gas void fraction and pressure ( Eq. 5 )
3. Assuming a given HR distribution that delivers a flow rate  $q$  during the time  $t$  and calculate the delivered fluid volume ( Eq. 6 ).
4. If the delivered fluid volume (Eq.6) complies with the gas compressed volume (Eq.5), then the design process is balanced and the iterative process is stopped. The design iterative process gives the dimensions and the distribution of the hydraulic regulators HR system.

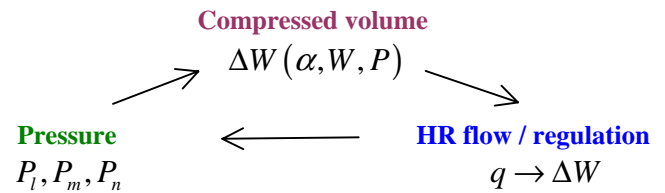


Figure 2. HR design procedure

Another effect caused by pressure gradient is the viscous-friction torque. The interference between rotor and stator is also presented in the paper [1]. In these conditions, the temperature becomes a mesure of friction torque, according to Bowden:

$$T_m \sim \Delta P_{n-m} \times f \times N \quad (\text{Eq. 7})$$

where  $\Delta P_{n-m}$  is defined as the differential pressure between the cavities  $n$  and  $m$ ,  $f$  is the friction coefficient equivalent to Newton dynamic viscosity , and  $N$  is the pump rotation velocity.

Since the pressure gradient is determined, the reliability parameters are under control and the temperature caused by gas compression ( thermo- hydraulic process ) and rotor-stator viscous - friction torque ( thermo-mechanical process) decreases .

Accordingly, the temperature is a measure of gas compression and viscous torque, which are both determinant factors of the reliability. We can then state that the *temperature ratio* is in fact a *reliability criterion* .

### The Experimental Program

We performed an extensive test program at the PCM's facility test rig and CREMHyG, the turbomachinery research and test laboratory in Grenoble, France. The objective of this experimental program was to determine the NPCP's ability to handle multiphase fluids and to enhance the reliability in real production conditions. That is: high gas void fraction (GVF= 0.9), operational pump head, velocity and flow rate.

Additionally, NPCP with different hydraulic regulators HR designs were evaluated in liquid and multiphase flow. We then compared with the original PCP and found a noticeable improvement in reliability performance of NPCP compared to traditional PCP. Figure 3 shows the pump test skid at CREMHYG. Since the temperature and pressure distributions are significant factors, measurements were carried out at multiple locations and different time intervals. The presented temperature values were measured during 20 minutes of stable multiphase flow conditions. In actual operation, long term response to thermal increase largely depends upon initial temperature gradient (system invariant) and surrounding factors. Flow meters on gas and liquid lines were used. Additionally, pump speed, delivered pressure and power were also recorded. The range of measured parameters is shown in Table 1.



Figure 3. The test rig and the measurement installation.

Pump characteristics	
NPCP Pump: PCP, PCM Oilfield 100 TP 600	
Flow rate = 108 m <sup>3</sup> /d (680 bpd)	
Head = 600 m (2000 ft); N= 500 rpm	
Experimental conditions	
Liquid: water, oil ;and Gas: air	
Flow rate (m <sup>3</sup> /d) = 10 to 60. Pressure = 0 to 40 bar	
GVF = 0 to 0.9, Oil viscosity (cPo) = 1200 (20°C) and 600 (30°C)	
Measured parameters	
Pressure = 21 sensors, Temperature = 10 sensors,	
Flow rate: liquid, gas.	
Pump velocity (rpm)	Power ( kw )

Table 1: Test conditions

### Comparison of NPCP and PCP

The method used to estimate the hydraulic regulators HR efficiency, the major difference between the NPCP and traditional PCP, can be expressed in term of the relationship

between pressure and temperature, for both thermal processes. The reliability criterion can then be obtained. Thus, we note by subscripts the mean parameters: 1 for NPCP and 2 for PCP. From *thermo- hydraulic* Equations 1 and 2, defining the gas compression, we derive the evaluation that compares NPCP and traditional PCP thermal performance, where  $p$  and  $t$  stand respectively for pressure and temperature gradients:

$$\frac{p_2}{p_1} \sim \frac{t_2}{t_1} \quad (\text{Eq.8})$$

The *thermo-mechanical* Equation 7, defining the viscous friction rotor-stator caused by the local high pressure gradient, gives a comparison of NPCP versus traditional PCP, as follows:

$$\frac{T_2}{T_1} \sim \frac{\Delta P_2}{\Delta P_1} \times \frac{f_2}{f_1} \times \frac{N_2}{N_1} \quad (\text{Eq. 9})$$

*Pressure and temperature in liquid flow.* Figure 4 shows the pressure distribution in liquid (oil) flow, for the original PCP and NPCP with distributed HR along the pump. Liquid pressure curve of PCP is the result of rotor-stator interference fit, particularly on the inlet sector.

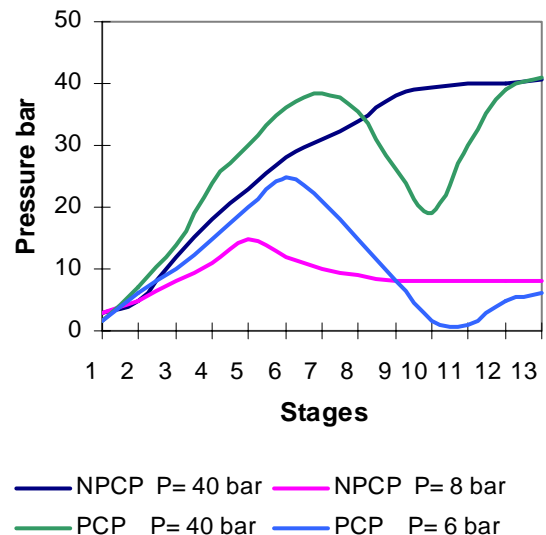


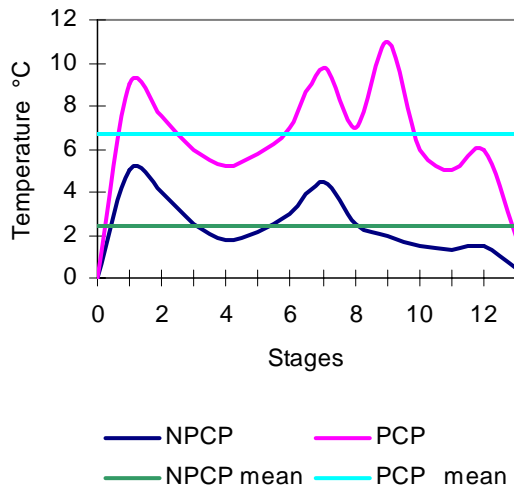
Figure 4. NPCP vs. PCP .Pressure distribution along the pump in liquid flow; P= 40 and 6 to 8 bar , N= 300 rpm .

Indeed, when the discharge pressure is low (0 to 6 bar) the pressure increases through the inlet stages ( from 1 to 7 ) . That is due to the interference fit which seals the cavities and a very small variation (0.2%) of incompressible liquid volume causes high pressure variation. On the other hand, the slippage is located on the discharge stages (P= 40 bar) and the actual pressure curve is uneven. A consequence is that the local pressure can reach the cavitation conditions and damage the elastomer; for example, in viscous oil flow, the very low

pressure of stages 9 to 12 ( Figure 4 ) may cause local gas cavitation .

In liquid flow, the HR recirculation between the cavities balances the pressure, and the NPCP pressure gradient becomes uniform. Moreover, the liquid volume variation of a cavity is compensated, thus there is no exaggerated pressure increase and that protects the stator. As for cavitation risk due to local low pressure, this will be definitely avoided by HR pressure regulation (Figure 4).

Since the pumped liquid is incompressible, the temperature ( Figure 5) is due only to *viscous friction* developed by the rotor – stator fit under the pressure gradient (Eq.9). As mentioned (Figure 4), the PCP uneven pressure distribution shows high local pressure gradient of  $\Delta P_2 = 7.7$  bar/stg, while the NPCP uniform pressure gradient is  $\Delta P_1 = 3$  bar/stg.



**Figure 5. NPCP vs. PCP. Temperature distribution along the pump in liquid flow. N= 300 rpm, P = 40 bar (see Figure 4).**

Furthermore, the mean temperatures (Figure 5) are  $T_1 = 2.4$  °C for NPCP and  $T_2 = 6.7$  °C for PCP. Thus, the NPCP-HR pressure gradient and temperature regulation (Eq. 9) is confirmed:

$$\frac{\Delta P_2}{\Delta P_1} = \frac{7.7}{3.0} = 2.6 \quad \text{and} \quad \frac{T_2}{T_1} = 2.8$$

Viscous coefficient  $f$  and velocity  $N$  were identical for NPCP and PCP tests . In fact, the temperature is a measure of the stator response to rotor viscous torque. NPCP temperature is 1/3 of PCP's, and that is due to decrease of NPCP pressure gradient.

Consequently, in liquid flow the experimental data show the NPCP reliability criterion (Eq.9) is significantly improved.

*Pressure and temperature in multiphase flow.* Therefore, both processes ,gas compression (Eq. 8) and viscous torque (Eq.9), do take place. The first discussed NPCP vs. PCP comparison shows very severe conditions: large amount of gas (GVF= 0.9 and flow rate corresponding to N= 300 rpm ) , high discharge pressure (P = 40 bar or equivalent two-phase pump head H= 4000 m; see Appendix ) and the two-phase mixture of oil – air. Even in this case, it is clear that the HR regulation changes significantly the pressure distribution. Figure 6 shows that the NPCP compresses the gas with 8 stages, instead of 3 discharge stages of traditional PCP. Therefore, the pressure

gradient ratio is  $\frac{p_2}{p_1} = \frac{8}{3} = 2.7$

There is no gas compression at the inlet stages (Figure 6), thus the temperature is due to *viscous-friction torque* (Eq.9). As the velocities and friction coefficients are similar , the temperature gradient ratio becomes (Figure 7;see mean

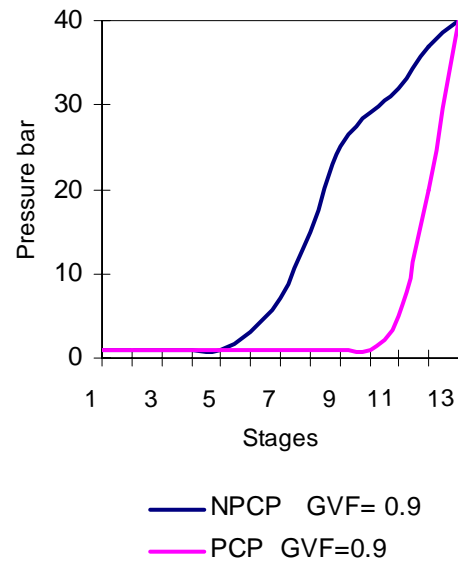
temperature upon inlet stages )  $\frac{T_2}{T_1} = \frac{7.5}{2.5} = 3$  , which is

equivalent to pressure gradient.

Since the *gas compression* takes place particularly on the discharge stages ( Figure 7), the temperature gradient ratio

(Eq. 8) is  $\frac{t_2}{t_1} = \frac{21}{6.3} = 3.3$ , which is also in relation to

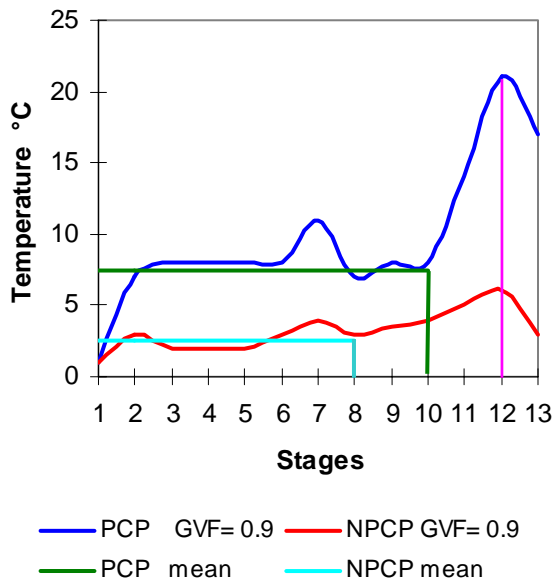
pressure gradient .



**Figure 6. NPCP vs. PCP. Pressure distribution along the pump in multiphase flow for GVF= 0.9, N= 300 rpm and P = 40 bar.**

Finally, we can conclude that:

- the NPCP system has low pressure gradient that results in reduced temperature developed by both processes (gas compression and viscous torque) and ,
- the *reliability criterion* of NPCP is 3 time better than that of the PCP.



**Figure 7. NPCP vs. PCP. Temperature in multiphase flow along the pump. GVF= 0.9, N= 300 rpm, P= 40 bar .**

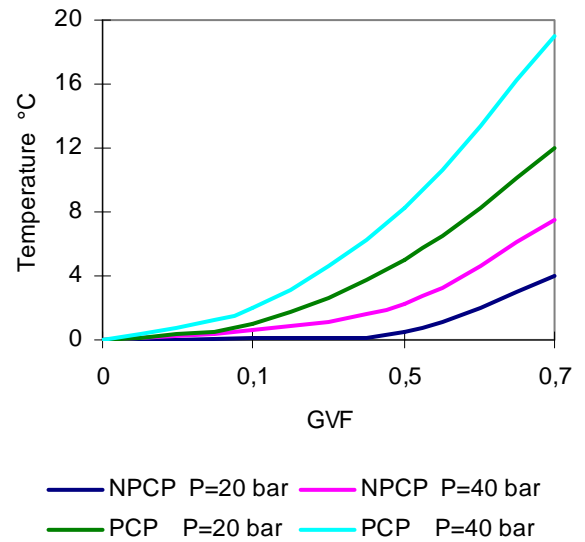
The previous case was dedicated to fixed production parameters: GVF, P and N. Thus, the question is: whatever the pumping conditions are, is the reliability criterion an invariant of the system?

To illustrate the hydraulic regulators HR response to gas content and delivered pressure variation, Figure 8 shows the two-phase temperature for various GVF and P, delivered pressure. Here, the two-phase temperature is the difference between the temperature developed in two-phase flow and liquid flow. Figure 8 shows the growth of two-phase temperature along the pump, between the inlet and discharge sections. Although, the two-phase mixture is water-air, the temperature ratio NPCP vs. PCP is coherent with previous results, i.e. the mean reliability criterion is roughly 3, independently of GVF and P.

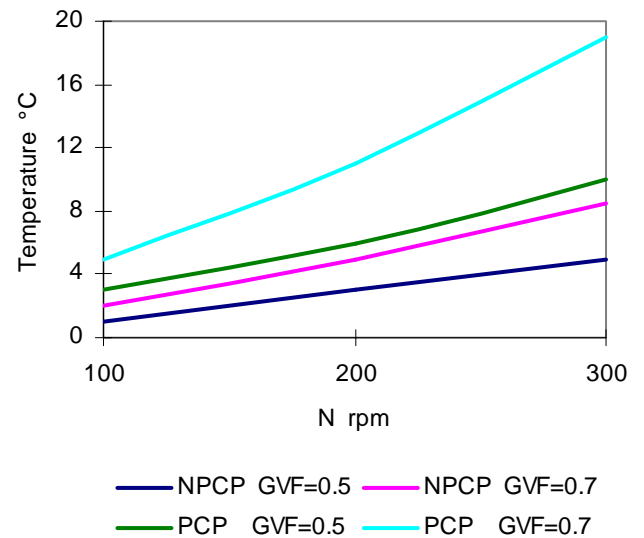
This result confirms the hydraulic regulator HR *self-regulation* ability. Equation 6 shows that the gas compressed compensation by HR flow rate  $q$  depends on pressure and time, so the HR works the time required to balance the pressure and compensate the cavity. Therefore, a HR designed for large GVF and high pressure will self-adapt the working time until the pressure is balanced, when  $q$  tends to 0. Thus, when the GVF and P are lower than in the design specifications, the HR will work less time but the pump will reach the similar value of reliability criterion.

Another parameter is the velocity N, which gives the gas flow rate. Figure 9 shows the two-phase temperature as a function of N for various GVF. As the gas flow rate linearly increases with N, so will do the two-phase pressure distribution gradient. Therefore, both thermal processes (gas compression and viscous torque) will be linear functions of N. For example, the temperature due to friction torque (Eqs.7 and 9) depends linearly of N, as is shown on Figure 9. In the same time, better pressure distribution of NPCP results in significant reduction of viscous torque (Eq.7). The NPCP reliability is

improved and consumed power is reduced. The NPCP mean reliability criterion is coherent with the mentioned values.



**Figure 8. NPC vs. PCP. Temperature for various GVF, delivered pressure P= 20, 40 bar and N= 400 rpm.**



**Figure 9. NPCP vs. PCP. Temperature function of pump velocity, P= 40 bar.**

## Conclusions

1. A New Progressing Cavity Pump (NPCP) that works with multiphase flow is described. NPCP is the result of hydraulic regulators HR, installed inside the rotor of actual PCP, between the cavities. The HR system allows the pump to control both reliability parameters and pump lift performance in multiphase flow.

2. The HR self-regulated system enhances the pump response in multiphase production: reduces the temperature due to gas compression and viscous torque, improves the reliability of stator elastomer, reduces the power consumption, avoids

cavitation risk in viscous flow and finally improves hydraulic performance ( delivered pressure and flow rate).

3. An extensive experimental program carried out on the original PCP and the NPCP enabled us to compare reliability and production performance. Based on thermo-hydraulic-mechanical processes, a simple reliability criterion is proposed.

4. The comparison NPCP and PCP shows that reliability criterion of NPCP is roughly 3 time that of the PCP. Tests confirm that this improvement level is valid for various gas content GVF, flow rates, delivered pressures and pump velocities .

5. The NPCP design includes HR section and the distribution along the pump.The analytical method is confirmed by the experimental results and we propose an iterative process to find the optimal design .

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**Nomenclature**

- HR: hydraulic regulator
- P: pressure, differential pressure
- T: temperature
- $GVF = \alpha$  : gas void fraction
- $\rho$  : gas density
- k: gas polytropic coefficient
- T: temperature
- Q: pump flow rate
- q: HR flow rate of recirculation
- W: cavity volume
- WG: gas volume of a cavity
- MG: gas mass
- f: rotor-stator friction coefficient
- N: pump rotational velocity
- $\omega$  : Hydraulic regulator flow section
- $\zeta$  :Hydraulic regulator pressure drop coefficient

**References**

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2. Mirza K, Wild A. Key Advantages of the Progressing Cavity Pump in Multiphase Transfer Applications. SPE 38812. Annual Technical Conference and Exhibition. San Antonio, Texas, USA, 5-8 October 1997.

**SI Metric Conversion Factors**

- 1 inch ( in ) = 2.54 cm
- 1 foot ( ft ) = 0.305 m
- 1 barrel (B) = 0.159 m<sup>3</sup>
- 1 bar = 10<sup>5</sup> Pa
- 1 psig = 6.89 K Pa
- ( 5/9 ) ( °F-32 ) = °C

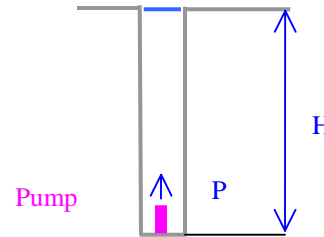
**Appendix . Two-Phase Pump Head**

The down hole pump pressure P is equivalent to two-phase head H :

$$H = \frac{1}{g} \int \frac{dP}{\rho_m} \quad ; \rho_m = GVF \times \rho_G + (1 - GVF) \rho_L$$

where  $\rho_m$  is the mixture density and  $\rho_G, \rho_L$  are the densities of gas and liquid phases. Therefore, the pump total head includes flow pressure drop and well depth. Consider the polytropic process relationship, which associates gas density and pressure

$$\frac{P}{\rho_G^k} = Ct$$



We can conclude that at the pump discharge pressure P the gas density reaches a maximum value and the pump head estimation becomes:

$$H > \frac{P}{g \rho_m}$$

Example:

P= 40 bar and corresponding water head  $H_{Liquid} = 400 \text{ m}$  ; Gas void fraction  $GVF = 0.9$

$$\rho_m \approx (1 - GVF) \times \rho_L = 0.1 \times \rho_L$$

$$H \approx \frac{P}{g \rho_m} \approx \frac{P}{0.1 \times \rho_L g} = 10 \times \frac{P}{g \rho_L} = 10 \times H_{Liquid} = 4000 \text{ m}$$

This approximation neglects the gas density and the liquid density is equivalent to water density; thus, the pump head H is underestimated.